Thermoelectric Modules: Recursive non-linear ARMA modeling, Identification and Robust Control

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Abstract—This work deals with the modeling and identification of thermoelectric modules. We propose models for both steady-state and unsteady-state dynamics based on recursive ARMA models for temperature and heat flux. The proposed models are convenient for simulation, control, electronic, and thermal engineers. They also help understanding the functionality of the heat pumps, and facilitate the solving of cooling/heating problems without the need for expertise in thermal engineering. The effectiveness of the models and the synthesized controller are assessed by simulation and experimentally.

I. INTRODUCTION

A thermoelectric cooler (TEC) consists of a multiple semiconductor junction connected electrically in series and put between two plates (fig. 1). These plates must, in the same time, be conductors of heat and good electrical insulators. Ceramic materials fulfill this particular property: one plate is thermally connected to a radiator, and the other to the object whose temperature is to be controlled. Thanks to the Peltier physical effect, a current through the junctions creates a temperature difference between the plates whose polarity and magnitude depend on those of the current. Relatively to the ambient temperature, it then becomes possible to heat the object or cool it down (via one TEC plate). Today’s technology allows temperature differences as high as 84°C, and cascading arrangements can produce even higher differences.

The Peltier devices are widely used today in many applications, from thermal stabilization to micro-refrigeration. TEC can be used where cooling or temperature control of an object is required. Generally, TEC is most often used:

1) in applications where an object needs to be cooled below the ambient temperature, or needs to be maintained at a constant temperature;
2) as a thermal-haptic display for virtual reality and telepresence systems [1] [2] [3]. Precisely in this case, we need to control the TEC in order to get fast and exact response both in temperature and in heat flux.

The main difficulty is to write accurate enough equations that relate the input current, driving the Peltier device, to the temperature of each Peltier’s side. These equations are hard to determine and a precise analysis of the electro-thermal Peltier effect is difficult; this is because of the nonlinear behavior of the Peltier device that act as a floating load for the voltage supply and the several phenomena that occur within it.

In [4], a SPICE compatible equivalent circuit of TEC has been devised. Equivalent circuit is a convenient tool for electronic engineers. This helps representing the problem in ‘electronic circuit language’ and facilitates the solving of cooling/heating problems without the need for expertise in thermal engineering. In [5], it has been shown that it is possible to describe the characteristics of a thermoelectric device with a second order discrete time ARMA model. The parameters of such recursive second order discrete-time model vary with the operating point which is defined by the bias excitation current.

This work aims at providing a more general model for TEC. We propose a non-linear dynamic representation based on a non-linear ARMA model for temperature control. We also propose a model for solving steady-state cooling/heating capacity and establish a model to describe the temperature/heat flow relation. These models should help users to synthesize control laws for achieving required performances.

![Fig. 1. Construction of a Peltier semi-conductor element. In practice several elements are generally connected in series (electrically), and in parallel (thermally).](image)

II. MODELING THE PELTIER PUMP

When an electric current flows through a circuit composed of two different conductors $n$—type and $p$—type (see fig. 1) heat is liberated at one junction and absorbed by the other one, depending on the direction in which the current is flowing: this is called the Peltier effect. The quantity of heat $P$ liberated per unit of time is proportional to the current [6]:

$$\frac{P}{t} = P_p = \pi I = \alpha T I$$

(1)

where $\pi$ [V](Volts) is the Peltier coefficient, $\alpha$ [V·K$^{-1}$] the Seebeck coefficient, $T$ [K] the absolute temperature and $I$
[A] the electric current. If an electric current flows in a homogeneous conductor in the direction of a temperature gradient $\frac{dT}{dx}$, heat will be absorbed or given out, depending on the material: this is the Thomson effect,

$$P_T = \tau \ I \ \frac{dT}{dx}$$  \hspace{1cm} (2)

where $\tau$ [VK$^{-1}$] is the Thomson coefficient and $x$ [m] the space variable. The direction in which the heat flows depends on: the sign of the Thomson coefficient, the direction in which the current flows, and the direction of the temperature gradient.

In the other side, the heat loss due to Joule effect occurs; they are assumed to be equal and uniformly distributed on the two sides. If an electric current $I$ flows in an isothermal conductor of resistance $R$, the Joule effect writes:

$$P_j = R \ I^2$$  \hspace{1cm} (3)

Because of heat conduction, heat also flows from the hot side (temperature $T_h$) to the cold side (temperature $T_c$), hence:

$$P_k = K \ (T_h - T_c)$$  \hspace{1cm} (4)

where $K$ is the thermal conductance. The heat capacity of the pump on the cold side (the steady-state cooling capacity) is:

$$P_c = \beta n \left\{ \alpha T_c I \pm \frac{\tau I (T_h - T_c)}{2c} - \frac{1}{2} R I^2 - K (T_h - T_c) \right\}$$  \hspace{1cm} (5)

where $c$ is the thickness of the Peltier component and $n$ the number of components. The heat capacity of the pump on the hot side (the steady-state heating capacity) is:

$$P_h = \gamma n \left\{ \alpha T_h I \pm \frac{\tau I (T_h - T_c)}{2c} + \frac{1}{2} R I^2 - K (T_h - T_c) \right\}$$  \hspace{1cm} (6)

Equations (5) and (6) represent the steady-state behavior of the Peltier device; transient behavior is more complex to catch. However transient behavior is required for real-time simulation and robust controller synthesis.

III. PROPOSED MODELS

Based on the previous steady-state models, we assume that the dynamic behavior of the TEC can be correctly described by the following discrete-time ARMA model [5]:

$$T(t) = -\sum_{i=1}^{n} a_i \ T(t-i) + \sum_{j=0}^{m} b_j \ I(t-j)$$  \hspace{1cm} (7)

where $a_i$ and $b_j$ are parameters depending on the current $I(t)$.

We propose recursive polynomial functions to describe the dependence of the parameters with the current. That is:

$$a_i = \sum_{k=0}^{p} a_{ik} \ I(t-i)^k$$  \hspace{1cm} (8)

$$b_j = \sum_{h=0}^{q} b_{jh} \ I(t-j)^h$$  \hspace{1cm} (9)

The complete description leads to the non-linear ARMA model:

$$T(t) = -\sum_{i=1}^{n} a_{ik} \ I(t-i)^k T(t-i) + \sum_{j=1}^{m} \sum_{h=0}^{q} b_{jh} \ I(t-j)^h I(t-j)$$  \hspace{1cm} (10)

which is a linear model of the unknown parameters $\theta$:

$$T(t) = \theta^T \varphi(t)$$  \hspace{1cm} (11)

where

$$\theta = [a_{ik}, \ {i = 1 \cdots n, \ k = 0 \cdots p}; b_{jh}, \ {j = 0 \cdots m, \ h = 0 \cdots q}]$$

is the vector of parameters of dimension $d$, and

$$\varphi(t) = [I(t-i)^k T(t-i), \ {i = 1 \cdots n, \ k = 0 \cdots p}; I(t-j)^h, \ {j = 0 \cdots m, \ h = 0 \cdots q}]$$

the regression vector of dimension $d = n(p + 1) + m(q + 1)$. For the relation between the temperature $T$ and the heat flux $Q$, we also proposed an ARMA model:

$$Q(t) = -\sum_{i=1}^{n} c_i \ Q(t-i) + \sum_{j=0}^{m} d_j \ T(t-j)$$  \hspace{1cm} (12)

where $c_i \ {i = 1 \cdots n}$ and $d_j \ {j = 0 \cdots m}$ are the model parameters to be identified.

IV. EXPERIMENTAL SET-UP

The experimental setup consists of two contact pads using a Peltier pump from MELCOR CorpTM of dimension $15mm \times 15mm \times 3.2mm$ (see fig. 2).

![temperature and heat flux sensor](image)

Fig. 2. View of the Peltier pump and the flow/temperature sensors disposition.

Each Peltier pump is embedded with a dissipater having high thermal conductivity, on the one face, and a Captec heat flux/temperature sensor on its other face. Silicon grease is used to decrease the contact resistance at the contact surface of the Peltier pump. Because of its reduced thickness, the thermal transfer within the sensor section is supposed to be linear and homogeneous. The current driving the Peltier pump is amplified. Voltage amplifiers are used for the flux/temperature sensors signals; current sensors measure what is actually provided to each device. The control is accomplished using a dSPACE setup.
V. MODELS IDENTIFICATION

A. Identification of the heating/cooling capacity models

The equations model (6) and (5) can be also rewritten:

\[
\begin{align*}
& P_h = \theta_h I_T + \theta_{h2} I^2 + \theta_{h3} I_T + \theta_{h4} I + \theta_{h5} = \theta^T_h \varphi_h \\
& P_c = \theta_c I_T + \theta_{c2} I^2 + \theta_{c3} I_T + \theta_{c4} I + \theta_{c5} = \theta^T_c \varphi_c
\end{align*}
\]

with

\[
\varphi_{h/c} = [T_{h/c}, I^2, T_{h/c}, I, 1]^T
\]

being the measure vectors and \( \theta_{c/h} \) the unknown vectors.

Models given by the equations (13) are linear functions of the unknown vectors.

Experiment with \( N = 14 \) and different currents values in the interval \([-1.5\text{A},+1.5\text{A}]\), has led to a set of measurements (current, temperature and heat flux) which illustrate the permanent behavior of the device. The elements of the vectors \( \theta_{h/c} \) can be determined by solving the following parameter optimization problem where:

\[
\hat{\theta}_{h/c} = \arg \min_{\theta_{h/c}} \left\{ \sum_{t=1}^{N} \left[ P_{h/c}(t) - \hat{P}_{h/c}(t) \right]^2 \right\}
\]

where \( N \) denotes the length of the measurements vectors. Using the linear Least Square Method (LSM) we have:

\[
\hat{\theta}_{h/c} = (\Phi^T_{h/c} \Phi_{h/c})^{-1} \Phi^T_{h/c} P_{h/c}
\]

where:

\[
\Phi_{h/c} = [\varphi_{h/c}(i), \ i = 1 \cdots N ]
\]

are measurement matrices. The estimated cooling/heating capacity by the identified model matches the real measurement (see fig. 3 and fig. 4). The estimated parameters are given in table I in the appendix.

B. Identification of the Temperature Dynamic Model

An open loop excitation is performed using a Pseudo Random Binary Sequence (PRBS) signal approximating a white noise signal with variable amplitude sampled at 0.01sec. This sequence is sent to the Peltier pump driving power. The same experiment is realized with the two Peltier pumps. The parameters of the model equation (10) can be determined by solving the following parameter optimization problem:

\[
\hat{\theta}_{LS} = \arg \min_{\theta} \left\{ \sum_{t=n}^{M} \left[ T(t) - \hat{T}(t) \right]^2 \right\}
\]

where \( \hat{\theta}_{LS} \) and \( \hat{T}(t) \) are the estimated parameters and the temperature respectively, \( M = 10000 \). The LSM algorithm is also used,

\[
\hat{\theta}_{LS} = (\Phi^T \Phi)^{-1} \Phi^T Y
\]

where \( Y = \{ T(t) , t = n \cdots M \} \).

The above parameter optimization problem can be improved by using the Recursive Least Square Method (RLSM):

\[
\hat{\theta}(t) = \hat{\theta}(t-1) + K(t-1) \varphi(t) \varepsilon(t)
\]

\( \hat{\theta}(t) \) is the estimated vector at \( t \), \( K(t) \) the Kalman matrix gain, and \( \varepsilon(t) \) the estimation error defined as:

\[
\varepsilon(t) = \frac{y(t) - \theta(t-1) \varphi(t)}{1 + \varphi(t)^T K(t-1) \varphi(t)}
\]

The Kalman gain update is computed as follows:

\[
K(t) = \frac{1}{\lambda_1} \left[ K(t-1) - \frac{K(t-1) \varphi(t) \varphi(t)^T K(t-1)}{\lambda_2 + \varphi(t)^T K(t-1) \varphi(t)} \right]
\]

\( \lambda_1 \) and \( \lambda_2 \) are the forget factors, \( K(0) \) the initial matrix gain (influences the convergence properties of the algorithm).

The fig. 5 illustrates the measured TEC temperature compared to the estimated temperature, the identified model is for \( n = p = m = q = 3 \). In the worst case, the error between real and estimated temperatures reached 0.3°C. The estimated parameters are given in table II of the appendix.

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Fig. 3. Estimated (line) and measured (cross) heat capacity.

Fig. 4. Estimated (line) and measured (cross) cooling capacity.

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C. Temperature/heat flux dynamic model

The measurement sequences performed in the previous section are also used for the identification of the parameters of the temperature/heat flux model. The LSM algorithm is used to get the optimal solution. The fig. 6 illustrates the measured TEC heat flux compared to the estimated one by the identified model with \( n = m = 3 \). The obtained results show good approximations of the real heat flux based on the previous measurement of the temperature. In the worst case, the error between the real and estimated heat flux reached 0.0025W. The estimated parameters are given in table III of the appendix.

VI. SLIDING MODE TEMPERATURE CONTROL

This section presents the design of a robust controller based on the sliding mode method for the identified TEC temperature model. Starting with temperature ARMA model, the equation (10) can be rewritten as follow:

\[
T(t) = -\sum_{i=1}^{n} a_i T(t - i) + \sum_{j=0}^{m} b_{j0} I(t - j) + \xi(I, T, t) \tag{18}
\]

where \( \xi(I, T, t) \) is the non-linear dynamic of the model. The simplified linear ARMA model is then derived as:

\[
T(t) = -\sum_{i=1}^{n} a_i T(t - i) + \sum_{j=0}^{m} b_{j0} I(t - j) \tag{19}
\]

From the equation (19) we get a discrete-continuous transformation using the zero order holder method:

\[
\begin{align*}
\frac{d^3 T(t)}{dt^3} + l_1 \frac{d^2 T(t)}{dt^2} + l_2 \frac{dT(t)}{dt} + l_3 T(t) &= m_1 \frac{d^2 I(t)}{dt^2} + m_2 \frac{dI(t)}{dt} + m_3 I(t) \tag{20}
\end{align*}
\]

From the equation (20), the state space model writes:

\[
\begin{align*}
\dot{x}(t) &= A x(t) + B u(t) \\
y(t) &= C x(t) + T(t)
\end{align*} \tag{21}
\]

where \( x(t) \) is the state vector, \( u(t) \) is the control input and \( y(t) \) is the output. The matrices \( A, B \) and \( C \) are defined as follow:

\[
A = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-l_3 & -l_2 & -l_1
\end{bmatrix}
\]
\[
B = \begin{bmatrix}
1 & 0 & 0 \\
l_2 & 1 & 0 \\
l_3 & l_2 & 1
\end{bmatrix}^{-1} \begin{bmatrix}
m_1 \\
m_2 \\
m_3
\end{bmatrix} = \begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{bmatrix}
\]
and\(^1\)
\[
C = [1 \ 0 \ 0]
\]

We define the sliding surface according to [7]:
\[
s(t) = e(t) + \lambda \ e(t)
\] (22)
where \(\lambda\) is a strict positive constant, and \(e(t) = y_d(t) - y(t)\) the output error and \(y_d(t)\) the desired state. Substituting the state space expression in the sliding surface equation we have:
\[
s(t) = [\dot{y}_d(t) + \lambda y_d(t)] - [\lambda x_1(t) + x_2(t) + \alpha_1 u(t)]
\] (23)
differentiating the sliding variable yields:
\[
\dot{s}(t) = [\dot{y}_d(t) + \lambda \dot{y}_d(t)] - [\lambda \dot{x}_1(t) + \dot{x}_2(t) + \alpha_1 \dot{u}(t)]
\] (24)
by substituting \(\dot{x}_1(t)\) and \(\dot{x}_2(t)\) expressions in equation (24), we have:
\[
\dot{s}(t) = [\ddot{y}_d(t) + \lambda \ddot{y}_d(t)]
\]
\[
- [\lambda x_2(t) + x_3(t) + (\lambda \alpha_1 + \alpha_2)u(t) + \alpha_1 \dot{u}(t)]
\] (25)
the equivalent control law \(u_{eq}(t)\) to achieve \(\dot{s}(t) = 0\) can be found as follow:
- let,
\[
g_1 = \alpha_1
\]
\[
g_2 = \lambda \alpha_1 + \alpha_2
\]
\[
g_3 = [\ddot{y}_d(t) + \lambda \ddot{y}_d(t)] - [\lambda x_2(t) + x_3(t)]
\]
and
\[
v(t) = \frac{g_1}{g_2} \dot{u}(t) + u(t)
\]
Substituting the last expressions in the equation (25), we found:
\[
v_{eq}(t) = \frac{g_3}{g_2}
\]
\(v_{eq}(t)\) is interpreted as the augmented equivalent control to achieve \(\dot{s}(t) = 0\) with the identified model parameters \(g_2\)
- as we do not know the exact model of the system, we introduce a correction term so that the following inequality is reinforced:
\[
\frac{1}{2} \frac{d}{dt} (s(t))^2 = s(t) \dot{s}(t) \leq -\eta \|s(t)\|^2
\] (26)
This inequality is the sliding condition, it ensures us that the distance to the sliding surface decreases along all the trajectories, with a minimal speed given by \(\eta\).

The augmented control law \(v_a\) is chosen to be:
\[
v_a(t) = v_{eq}(t) - k \ \text{sgn}(s(t))
\]
\(^1\)Experiments on the TECs show that \(\alpha_i \in [1, 2, 3] \neq 0\).

now, we can write the equation (25) as follows:
\[
\dot{s}(t) = g_3 - g_2 \ v(t)
\]
\[
= g_3 \left[1 - \frac{g_2}{g_2}\right] + g_2 \ k \ \text{sgn}(s(t))
\]
since:
\[
s(t) \ \dot{s}(t) = s(t) \left[g_3 \left[1 - \frac{g_2}{g_2}\right] + g_2 \ k \ \text{sgn}(s(t))\right]
\]
\[
\leq \left[g_3 \left[1 - \frac{g_2}{g_2}\right] + g_2\right] \ |s(t)|
\] by choosing the sliding gain \(k\), such as:
\[
g_3 \left[1 - \frac{g_2}{g_2}\right] + g_2 \leq -\eta
\] (27)
the sliding condition (26) is satisfied. The control input \(u\) expression is then deduced by solving the differential equation \(\left(\frac{g_2}{g_2}\right) \ \dot{u}(t) + u(t)\) \(- v_a = 0\), for which a simplified solution is:
\[
u(t) = u_{eq}(t) + k \ \text{sgn}(s(t))
\] (28)
where
\[
u_{eq}(t) = \delta \ \text{exp}\left(\frac{-\hat{g}_2}{\hat{g}_1}\right) + \frac{g_3}{g_2}
\] (29)
\(\hat{g}_1\) is the estimated value of \(g_1\).

The synthesized controller was first validated by computer simulation using the temperature identified model described by the equation (10) to track an amplitude varying reference. The controller parameters are initialized with \(k = -0.9\), \(\lambda = 250\) and \(\delta = 20\). The fig. 7 illustrates the simulation result.

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**Fig. 7.** Reference temperature (dashed) and estimated temperature (solid) of the Peltier pump.

The controller was then implemented experimentally where the reference signals was varying from \(\pm 10\degree\ C\). The states \(x_2\) and \(x_3\) were computed using observer. The reference and the measured TEC temperatures are shown in fig. 8.
VII. Conclusion

In this work we proposed three models describing the thermoelectric modules dynamics in steady-state and transient phases. The proposed models have been identified using the LSM and RLSM algorithms and assessed by simulation and experimentally. They prove to be efficient for analyzing the TEC heat flux and temperature behavior using linear and non-linear recursive ARMA equations.

We also presented a methodology for sliding mode temperature control design based on the identified parameters and validated the synthesized controller. The proposed TEC models and the developed controller will be implemented on a haptic system for virtual reality and telepresence applications.

References