Theoretical and Experimental Study of a Heat Transfer Model for Thermal Feedback in Virtual Environments

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Abstract—This paper presents a theoretical and experimental study of a model describing the heat exchange between a fingertip and any given touched object. This model is solved and implemented on a thermal display incorporated into a haptic device. Thermal feedback can assist in object identification, representation or in the creation of a complete haptic signature of a given object. Alternatively, thermal feedback could be used as a sensory substitute or adjunct for visual or tactile feedback. Experimental results validating the proposed thermal model are evaluated when touching virtual objects in virtual environments.

I. INTRODUCTION

The thermal sensation, permeating and accompanying our tactual impressions, influences considerably our capability in distinguishing several objects in our surrounding world by the simple act of touching. Thermal sensation plays an important role and is inherent to our haptic modality. Despite the various possible applications in telepresence and virtual reality systems, thermal feedback has not been extensively investigated relatively to force and other tactile feedbacks. Few thermal displays exist that are really able to reproduce high fidelity thermal sensations. On the other hand, no device or sensor developed so far is able to quantify our thermal perception in a reproducible way.

A brief survey in thermal rendering shows that the human thermal perception is still not well/totally understood. Most of reported well established knowledge deals with thermal threshold perception that allows distinguishing between warm and cold feeling [1] [2], while other researches deal with the human capability in materials discrimination using thermal cues [3] [4]. Many thermal displays have been developed for the stimulation of the human finger in contact with a virtual or a remote real object. Almost all thermal displays consist of a heat source, a Peltier pump or a power heater, embedded with a set of temperature sensors. The control technique reproduces pre-measured and pre-computed temperature profiles on a thermal pad in contact with the human finger [5] [6] [7].

Some recent approaches in the area of haptics, use physiological composition of the human finger (bone, blood and skin) and thermo-electricity laws to model and explain the heat exchange occurring in each part of the finger when touching an object. In [8], an artificial robotic finger is developed for comfort research in car industry; the device is used for measuring the contact point temperature by mean of sensors placed in an elastomer having thermal properties close to that of the human finger. The authors in [9] present a model for the heat transfer occurring between the finger and a material. This model is based on electric analogy and takes into account various phenomena like the applied pressure by the finger, the speed of the blood circulation, the surface state of the skin and the material. This study is developed for computing a thermal feedback model in virtual reality applications. In [10], authors presented temperature rendering based on contact temperature prediction and finger effusivity measurement. In [11], the heat conduction between skin and object are modeled by ordinary differential equations. Three different layers of tissue (epidermis, dermis and endodermis) are distinguished within the finger. The work of [12] proposes a prediction model for contact heat flux and temperature based on an infrared camera during interactions phases. An infrared thermal measurement system is also designed in [13] so that it can be used to evaluate the performance of a thermal model implemented in a thermal display. This measurement system requires that the contact material be transparent in both the infrared and visible spectrum. It is therefore designed so that skin temperature can be measured more accurately during contact, but not as a system that would be integrated into a haptic interface.

Most of thermal feedback methods use a direct temperature control scheme and do not produce satisfactory thermal sensation since the human thermal perception relies on the heat transfer rate, and not on the sole temperature. An exception is the work presented in [14] where the thermal feedback is based on a bilateral coupling scheme between the heat flux and the temperature using a four channel coupling scheme [15]. Also, the authors in [16] propose a finite-difference calculation of the heat exchange and they combined the thermal display with a kinesthetic device. However the reproduction of the thermal behavior exhibited weak tracking performances because of the control loop gain tuning and latencies due to the sensor and the computation cost of the model.

In the present work, a theoretical and experimental study of a model describing the heat transfer which occurs when a finger touches an object is made. Models of the heat exchange are developed and implemented on a thermal display incorporated to a haptic device. This model accounts for the thermal dynamics of the finger, the sensor and the material by considering their thermal properties. Experimental
results validating the proposed thermal model are evaluated when touching virtual objects in virtual environments. The developed interface is used for thermal rendering and thermal identification of virtual touched object.

II. HEAT EXCHANGE MODELING

A. Basic experimental thermal display setup

Fig. 1 shows the experimental setup of the thermal display. It consists of a contact pad using one Peltier pump from MELCOR Corp™ of dimension 15mm×15mm×3.2mm as a thermal display. The temperature and the heat flux evolutions are measured thanks to a heat flux sensor with an integrated type T thermocouple, the sensor flux sensitivity is about 1.26μV/(W/m²). The sensor includes sensitive element assembled between two thin layers of a rectangular form with dimension of 15mm×15mm×420μm and is closed up to the Peltier pump. The thermocouple sensitive element is positioned at the center of the sensor between two conductive plates made of copper. From a performance standpoint a thermocouple measurement is stable over time. Also, computing the temperature from thermocouple voltage is a straightforward one-step calculation, making it simple. The Peltier pump is having a dissipater with high thermal conductivity in the one face, and a heat flux/temperature sensor on its other face.

Thermal contact between the heat flux sensor and the surrounding material is critical to achieve reliable heat flux measurements. Without a tied contact, heat likely flows around the sensor instead of through it, which biases the measurements. Eliminating air gaps around the sensor is the most common precaution that must be taken to maintain good thermal contact. Therefore, silicone grease is used to decrease the contact resistance at the contact surface of the Peltier pump.

A force sensor is introduced in order to measure the forces applied by the operator finger on the thermal display. The controller bloc is implemented on a host computer and it drives two separated power-amplifiers, each one for the Peltier pump, through a dSPACE setup. Due to the sensors sensitivity to noises, the measured temperature and heat flux signals are filtered by a 2nd-order low-pass digital filter.

B. Thermal Interaction Modeling

The quantitative characterization of heat transfer, in general, and that of the conduction heat transfer, in particular, relies on the evaluation of the heat transfer rate. When three objects made of different materials with different initial temperatures \(T_{1,i}\) and \(T_{2,i}\), respectively are made in contact, Fig. 2, the temperature evolution during the transient contact phase can be expressed using the Fourier’s law [17]. For the 1D case study, heat transfer conduction process can be expressed by the following system of partial differential equations:

\[
\frac{\partial^2 T_1(x,t)}{\partial x^2} = \frac{1}{\alpha_1} \frac{\partial T_1(x,t)}{\partial t} \tag{1}
\]

\[
\frac{\partial^2 T_2(x,t)}{\partial x^2} = \frac{1}{\alpha_2} \frac{\partial T_2(x,t)}{\partial t} \tag{2}
\]

\[
\frac{\partial^2 T_3(x,t)}{\partial x^2} = \frac{1}{\alpha_3} \frac{\partial T_3(x,t)}{\partial t} \tag{3}
\]

Here, subscripts 1, 2 and 3 denote respectively the operator finger, the sensor and the material. \(\lambda_k\) [W/m.K] is the thermal conductivity, \(\rho_k\) [kg/m³] is the density and \(c_k\) [J/kg.K] is the specific heat, \(\alpha = \lambda_k/\rho_ck_k\) [m²/sec] is the thermal diffusivity that characterizes the thermo-physical properties of the finger, the sensor, and the material, during contact.

Mathematically, the macroscopic diffusion equation of heat transfer is a partial differential equation in both space and time. Consequently, the heat transfer problem is a well-posed problem in the Hadamard sense. That is, there exists a solution, which is unique and depends continuously on the boundary conditions, if consistent initial conditions (ICs) and boundary conditions (BCs) for the temperature are prescribed, and if the heat sources definitions are specified [18] under the hypothesis of semi-infinite body for both the finger and the material. That is:

\[
\lambda_1 \frac{\partial T_1(0,t)}{\partial x} = \lambda_2 \frac{\partial T_2(0,t)}{\partial x} \tag{4}
\]
\[ \lambda_1 \frac{\partial T_1(0,t)}{\partial x} = h \left( T_1(0,t) - T_2(0,t) \right) \quad (5) \]
\[ \lambda_2 \frac{\partial T_2(-L,t)}{\partial x} = \lambda_3 \frac{\partial T_3(-L,t)}{\partial x} \quad (6) \]
\[ T_2(-L,t) = T_3(-L,t) \quad (7) \]

\( L \) [m] represents the thickness of the sensor and \( h \) [W/K.m²] represent the inverse of the thermal contact resistance. An important variable introduced through equation (5), is the contact thermal resistance. It is common to most thermal applications using different connected materials and is due to the real, imperfect mechanical contact between the two parts of a composite structure. The heat transfer is then accompanied by a supplementary temperature drop at these interface levels, through combined convection and/or radiation transfer.

By replacing \( T_1 = T_{1,i} \), \( T_2 = T_{2,i} \) and \( T_3 = T_{3,i} \) (with these new variable, initial conditions are zero), and introducing Laplace transforms to equations (1), (2) and (3) we have:

\[ \frac{\partial^2 \theta_1(x,s)}{\partial x^2} = \frac{s}{\alpha_1} \theta_1(x,s) \quad (8) \]
\[ \frac{\partial^2 \theta_2(x,s)}{\partial x^2} = \frac{s}{\alpha_2} \theta_2(x,s) \quad (9) \]
\[ \frac{\partial^2 \theta_3(x,s)}{\partial x^2} = \frac{s}{\alpha_3} \theta_3(x,s) \quad (10) \]

where \( \theta_k(x,s) \) represents the Laplace transform of \( T_k(x,s) \) for \( k = 1, 2 \) and 3, and \( s \) is the common Laplace complex variable.

A trivial general solution is to take:

\[ \theta_1(x,s) = A_{11}e^{-p_1x} + B_{11}e^{p_1x} \quad x \geq 0 \]
\[ \theta_2(x,s) = A_{22}e^{-p_2x} + B_{22}e^{p_2x} \quad -L \leq x \leq 0 \]
\[ \theta_3(x,s) = A_{33}e^{-p_3x} + B_{33}e^{p_3x} \quad x < -L \]

Where: \( p_1 = \sqrt{\frac{\alpha_1}{\rho_1}}, \quad p_2 = \sqrt{\frac{\alpha_2}{\rho_2}}, \quad p_3 = \sqrt{\frac{\alpha_3}{\rho_3}}, \quad A_1, B_1, A_2, B_2, A_3, B_3 \) are to be computed. The temperature \( T_1 \) gets a finite value when \( x \to +\infty \) and the temperature \( T_3 \) gets a finite value when \( x \to -\infty \). So that: \( B_1 = 0 \) and \( A_3 = 0 \).

The Laplace transform of the equations (4), (5), (6) and (7) gives:

\[
\begin{align*}
\lambda_1 p_1 A_1 - \lambda_2 p_2 A_2 + \lambda_3 p_2 B_2 & = 0 \\
(h + \lambda_1 p_1) A_1 - h A_2 - B_2 & = h L \frac{\partial T_1}{\partial x} \\
\lambda_2 p_2 e^{p_2 x} A_2 - \lambda_2 p_2 e^{-p_2 x} B_2 + A_3 p_3 e^{-p_3 x} B_3 & = 0 \\
e^{-p_2 x} A_2 + e^{-p_2 x} B_2 - e^{-p_3 x} B_3 & = 0
\end{align*}
\]

The resolution of that system leads to:

\[ A_1 = \frac{A_{10} + A_{11}e^{-2L\sqrt{\frac{\alpha}{\rho_1}}}}{s \left( C_1 + C_2 e^{-2L\sqrt{\frac{\alpha}{\rho_1}}} \right)} \quad (13) \]
\[ A_2 = \frac{s}{A_{30} e^{-2L\sqrt{\frac{\alpha}{\rho_2}}}} \quad (14) \]

\[ B_2 = \frac{B_{20}}{s \left( C_1 + C_2 e^{-2L\sqrt{\frac{\alpha}{\rho_1}}} \right)} \quad (15) \]
\[ B_3 = \frac{B_{30} e^{-2L\sqrt{\frac{\alpha}{\rho_2}}} + B_{31} e^{-2L\sqrt{\frac{\alpha}{\rho_1}}}}{s \left( C_1 + C_2 e^{-2L\sqrt{\frac{\alpha}{\rho_1}}} \right)} \quad (16) \]
\[ \theta_1(x,s) = \frac{A_{10} e^{-\left(\frac{\sqrt{\alpha}}{\sqrt{\rho_1}}\right) \sqrt{x}} + A_{11} e^{-\left(\frac{\sqrt{\alpha}}{\sqrt{\rho_2}}\right) \sqrt{x}}}{s \left( C_1 + C_2 e^{-2L\sqrt{\frac{\alpha}{\rho_1}}} \right)} \quad (17) \]
\[ \theta_2(x,s) = \frac{A_{20} e^{-\left(2L + \sqrt{\alpha}\right) \sqrt{x}} + B_{20} e^{-\left(\sqrt{\alpha}/\sqrt{\rho_2}\right) \sqrt{x}}}{s \left( C_1 + C_2 e^{-2L\sqrt{\frac{\alpha}{\rho_1}}} \right)} \quad (18) \]
\[ \theta_3(x,s) = \frac{B_{30} e^{-\left(\sqrt{\alpha}/\sqrt{\rho_2}\right) \sqrt{x}} + B_{31} e^{-\left(\sqrt{\alpha}/\sqrt{\rho_1}\right) \sqrt{x}}}{s \left( C_1 + C_2 e^{-2L\sqrt{\frac{\alpha}{\rho_1}}} \right)} \quad (19) \]

where:

\[ A_{10} = h B_2 \left( T_{2,i} - T_{1,i} \right) \left( B_3 + B_2 \right) \]
\[ A_{11} = h B_2 \left( T_{2,i} - T_{1,i} \right) \left( B_3 - B_2 \right) \]
\[ A_{20} = h B_1 \left( T_{2,i} - T_{1,i} \right) \left( B_3 + B_2 \right) \]
\[ B_{20} = h B_1 \left( B_3 + B_2 \right) \left( T_{1,i} - T_{2,i} \right) \]
\[ B_{30} = \frac{B_1 B_2 B_3}{B_1} \left( B_3 - B_2 \right) \left( T_{1,i} - T_{2,i} \right) \]
\[ B_{31} = \frac{B_1 B_2 B_3}{B_1} \left( B_3 + B_2 \right) \left( T_{1,i} - T_{2,i} \right) \]
\[ C_1 = \left( h \left( B_3 + B_2 \right) + B_1 B_2 \sqrt{5} \right) \left( B_3 + B_2 \right) \]
\[ C_2 = \left( h \left( B_3 - B_2 \right) + B_1 B_2 \sqrt{5} \right) \left( B_3 - B_2 \right) \]

\[ B_k = \sqrt{\lambda_2 \rho_2 C_1} \quad \text{[W}/\sqrt{\text{m.m².K}}\text{]} \text{ represents the thermal fusivity of the finger (} k = 1 \text{), the sensor (} k = 2 \text{) and the material (} k = 3 \text{).} \]

Sight the complexity of the resulting function, the inverse Laplace transform of the function is calculated numerically using the simple precision Stehfest-method \((N = 10)\) [19].

\[ T_k \left( x,t \right) = \frac{\ln(2)}{t} \sum_{j=1}^{N} \psi_1 \left( x, \frac{\ln(2)}{t} \right) + T_{k,i} \quad (20) \]

where:

\[
V = \{0.0343, 0.0333, 0.0333, 1279.000076, -15623.66689, 84244.16946, -23695.7129, 37591.6923, -30071.6923, 164062.5128, -32812.50256\};
\]

### III. Experiment and Discussion

**A. Simultaneous determination of the thermal contact resistance and the thermal properties of the sensor**

For thermal feedback experiments, five different materials are selected (table I). These materials are characterized by their different thermal properties with equal volume. The
The thermal properties of the objects are smoothened to lower roughness and ensure a minimal thermal contact resistance with a constant pressure force.

Experiments were performed with five persons. Before each experiment, the temperatures of the index finger of the subject and the material are measured thanks to an infrared thermometer. Subjects were instructed to keep their index fingertip on the support and to apply a force of $\approx 10$N – of course we instruct the users not to focus on this issue; we wanted to get a similar contact surface in most cases, although this does not have a noticeable influence on the variation of temperature at the moment of contact [12] [13].

The temperature evolution and the exchanged heat flux are recorded during 60 seconds, a sample profile is presented in Figs. (3) and (4) for $T_1, i = 36^\circ C$ and $T_2, i = 24.5^\circ C$. The difference between the heat flux profiles and the temperature profiles is apparent. Note that the temperature variation is conversely proportional to the thermal effusivity of the materials at the opposite of the exchanged heat flux.

Typical Peltier pump temperature controller uses PID. Although the PID controller is useful, it suffers from a dependence on the ambient temperature condition. The differential term attempts to measure the heating/cooling rate of the Peltier pump to overcome this problem; but being a derivative, it is inherently unstable for fast temperature variation; this is most often the case when having an instant contact between two materials. But, heat flux sensor offers an elegant alternative to the differential term because it measures directly the heating/coupling rate in the Peltier pump and does not depend on the thermal ambient conditions and perturbations.

The PID controller function using the heat flux sensor essentially replaces the derivative term with the sensor output, as follow:

$$u(t) = K_p e(t) + K_i \int e(t) dt + K_Q Q$$

where $K_p$ and $K_i$ are respectively the proportional and integral gains, $e(t)$ is the temperature error, $Q$ the heat flux and $K_Q$ the heat flux factor. The use of the heat flux term effectively predicts where the temperature is going and allows a feed-forward control of the Peltier device [20].

In our previous work [21], we performed modeling and identification of thermoelectric modules for both steady-state and unsteady-state dynamics that are based on recursive
ARMA models for temperature and heat flux. These models proved to be efficient and convenient for simulation and control.

C. Including the thermal display to a haptic interface

In order to be able including the thermal display to a haptic interface and virtual reality application, we use a PHANTOM Omni haptic device coupled to a planar XY table. The thermal display is incorporated to the mobile part of the planar table, (see Fig. 5). The PHANTOM is used only as an input device which provides the 3d position of the finger. The mobile part is controlled to track the position XY of the PHANTOM device. A virtual world including virtual material is simulated in order to provide visual rendering. The thermal behavior of each material is generated using the model developed in the equation (18) and the parameters identified in Table II including the thermal properties of the sensor. The temperature is controlled according to the equation (21) for \( x = -L/2 \). The simulation algorithm is quite simple: the fingertip’s position is given by the PHANTOM (since the finger is attached to its handle) and is sent to the virtual environment. A collision detection checks for the proximity distance between the virtual finger model and the virtual objects that are used in the experiment. Once a contact is determined, the touched material is identified and the predicted temperature of the sensor is computed by our model (that is \( T_2(t,x) \)). This temperature is then sent as desired temperature to be displayed to the user.

Fig. (6) shows that the temperature profiles are reproduced with accuracy in comparison to Fig. (3) under the same initial conditions. Note that the same experimental object temperature is reproduced under the operator finger which preludes good thermal sensations.

In the contrary to temperature, Fig. (7) shows that the flux profiles are slightly different in amplitude in comparison to Fig. (4). The reasons have two origins:

1) we did not impose a 10N force to be maintained (which may influences the contact area in the real experiment) and, more importantly,

2) the feed-forward controller at the master site is made in a nearly unilateral way and based only on temperature servoing. This results are thus in total accordance with remarks made in [14].

In the future, the virtual model will be adapted to accounts for temperature and flux evolutions given directly from the sensor and enriched to include a good estimate of the contact resistance from the real measure.

In perceptual term, experiments show that the operator is more sensitive to temperature drop and heat flux changes when the contact is established. Materials with large thermal conductivity/effusivity (metals) are found to be easier to discriminate compared to object having smaller one, especially in the case of consecutive contact (small or no thermal adaptation of the finger) and this demonstrates a satisfactory accordance as in the case of consecutive contact with real object. Materials with relatively similar heat conductivity/effusivity are difficult to be distinguished. The fact that the thermal display reproduces the touched material behavior in contact with the finger, improves the thermal interaction realism.
IV. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

This research presents an analytical method to compute temperature evolution when contact occurs between a finger and a material taking into account the influence of the sensor thermal properties and thermal contact resistance. Experiments show that the drop of the temperature of the finger depends on the thermal effusivity of the materials and the initial conditions. The temperature profiles are reproduced perfectly when touching a virtual material so that realistic thermal sensation (from a signal viewpoint) is guaranteed by our developed model and a feed-forward control scheme. We expect that the proposed thermal feedback contributes to the generation of realistic haptic signals that would increase the operator’s immersion feeling and assist the operator in material identification in virtual environment. Alternatively, thermal feedback could be used as a sensory substitute or adjunct for visual and/or tactile feedback.

B. Future Works

Although the obtained results are satisfactory for thermal rendering, some points must be further investigated, namely:

- Evaluation of the haptic/thermal interface using psychophysical analysis.
- Including thermal feedback in telepresence applications based on the developed models.
- Development of a compact solution for kinesthetic/thermal rendering in interactive virtual simulation and telepresence.
- Using a bilateral coupling scheme between the thermal rendering and the simulated thermal exchange.

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